whereas the energy losses are

$$W_{\text{walls}} = R_s \int_{\text{walls}} | H(r) \times i_z |^2 dS \qquad (29)$$

where R_s is the surface resistance of the walls and is equal to $R_s = (\pi f \mu / \sigma)^{1/2}$, f being the frequency, μ the permeability, and σ the conductivity. The energy losses in the cavity are

$$W_{\text{cav}} = \frac{\omega \epsilon_1 \tan \delta}{2} \int_{\text{cav}} | E_1(r) |^2 dr. \qquad (30)$$

Eqs. (27)–(30) can be evaluated for the TE_{onl} mode field components given in Appendix I and the result manipulated to give tan δ, the dielectric loss tangent, in terms of known parameters. The result is

$$\tan \delta = \frac{1}{Q_0}\left[1 + \frac{1}{\kappa} F(\alpha) G(\beta)\right] - \frac{l^2 R_s}{2\pi f^3 \mu^2 \kappa \epsilon_0 L^3}\left[1 + F(\alpha) G(\beta)\right] \qquad (31)$$

where

$$F(\alpha) = J_1^2(\alpha_n)/[J_1^2(\alpha_n) - J_0(\alpha_n) J_2(\alpha_n)]. \qquad (32)$$

and

$$G(\beta) = [K_0(\beta_l) K_2(\beta_l) - K_1^2(\beta_l)]/K_1^2(\beta_l). \qquad (33)$$

Graphical plots of $F(\alpha)$ vs α and $G(\beta)$ vs β are given in Figs. 5 and 6, respectively.

Summary of Measurement Techniques of Parametric Amplifier and Mixer Noise Figure*

R. D. HAUN, JR.†, MEMBER, IRE

Summary—**Expressions are derived for the noise factor of a frequency mixing circuit under two different operating conditions: 1) single-sideband operation with input only in a band of frequencies at ω_1; and 2) double-sideband radiometer operation with incoherent inputs in the bands both at frequency ω_1 and at $\omega_2 = \omega_3 - \omega_1$. In both cases, the output is taken only at ω_1.**

It is shown that the noise figure for radiometer double-sideband operation is not always 3 db less than for single-sideband operation. It is also shown that it is possible to obtain an output signal-to-noise ratio which is greater than the input signal-to-noise ratio for coherent double-sideband operation.

Methods are analyzed for measuring the effective noise temperature of this circuit by using a broad-band noise source.

I. Introduction

THE AUTHOR of this paper has recently had occasion to attempt measurements of the noise figure of a quasi-degenerate parametric microwave amplifier.[1] During the course of this experiment, it was quite surprising to find that the measured values of noise figure were less than 3 db, although a simple theory[2,3] (which was believed to be fairly well understood) predicted that the noise figure of this device should always be greater than 3 db. It was soon realized that this apparent anomaly had come about because the method of measurement (with a broad-band noise tube) gave the radiometer double-sideband rather than the single-sideband figure, whereas the theoretical calculations were for single-sideband operation only.

Further reflection on this problem also revealed that it is possible to obtain an output signal-to-noise ratio which is greater than the input signal-to-noise ratio with the parametric amplifier in double-sideband operation.

Searching the literature on noise figure for analogous situations, one finds that confusion exists on this point of single- vs double-sideband noise figure. Uenohara[4]

* Received by the PGMTT, September 14, 1959; revised manuscript received, February 17, 1960.

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[1] R. D. Haun, Jr. and T. A. Osial, "Measurements on a 4.6 kmcps Diode Parametric Microwave Amplifier," Westinghouse Electric Corp., Pittsburgh, Pa., Scientific Paper No. 6-41003-4-P1, June, 1959; unpublished.

[2] R. D. Haun, Jr., "Small Signal Theory of Microwave Parametric Amplifiers and Up-Converters Using High-Q-Non-Linear Reactances," Westinghouse Electric Corp., Pittsburgh, Pa., Scientific Paper, No. 8-1047-P2, November 3, 1958; unpublished.

[3] H. Heffner and G. Wade, "Gain, bandwidth, and noise characteristics of the variable parameter amplifier," *J. Appl. Phys.*, vol. 29, pp. 1321–1331; September, 1958.

[4] G. H. Herrman, M. Uenohara, and A. Uhlir, Jr., "Noise figure measurements on two types of variable reactance amplifiers using semiconductor diodes," Proc. IRE, vol. 46, pp. 1301–1303; June, 1958.

refers to this problem and simply states that the single-sideband noise figure F_{SSB} is 3 db greater than the double-sideband noise figure F_{DSB} for a quasi-degenerate parametric amplifier. Heffner[5] states that F_{SSB} is at least 3 db greater than F_{DSB} in a parametric amplifier. Cohn[6] shows that if the power gain of the device is the same for an input at each of the two sidebands, the double-sideband noise figure is 3 db less than the single-sideband value; however, he does not consider the case where the gain is different for inputs at different sideband frequencies.

It is the purpose of this paper to treat this problem in detail in the hope that it will resolve some of the confusion, so that different workers in the field of parametric amplification can specify unambiguously the noise properties of a given device.

A parametric amplifier or a nonlinear resistance mixer can be used in three ways, each of which will be discussed in the following section:

1) *Single-sideband operation* in which signal power flows into the network only in the frequency band at ω_1 and the output of the network is taken only in the band at ω_1;

2) *Double-sideband radiometer operation* in which incoherent signals are fed into the network in the bands both at ω_1 and ω_2, and the output of the network is taken only in the band at ω_1;

3) *Coherent double-sideband operation* where, for each input in the band at ω_1, there is coherent input (*i.e.*, an image of the input at ω_1) in the band at $\omega_2 = \omega_3 - \omega_1$ and the output of the network is taken only in the band at ω_1.

II. Definitions of Noise Quantities for Parametric Amplifiers and Mixers

The noise factor F of a network is defined as[6–8]

$$F = \frac{N_{\text{out}}}{GN_{\text{in}}} \tag{1}$$

where

N_{in} = the available input thermal noise power in a bandwidth B from a room temperature (290°K) resistor connected to the input terminals of the network;

N_{out} = the total noise power in the bandwidth B which flows out of the network toward a load (assuming the input is connected to the room temperature resistor giving rise to N_{in});

G = the power gain of the network for incoherent signals and is defined as

$$G = \frac{N'_{\text{out}}}{N_{\text{in}}}; \tag{2}$$

N'_{out} = that portion of the total output noise power which results from the input N_{in}.

It will be noted that the definitions of F and G given here contain the somewhat arbitrary condition that N_{in} be the available power from a room temperature source connected to the input of the amplifier. These definitions have a definite convenience in the microwave case because then the available power is simply the power which flows down the input transmission line toward the network, and G is the ratio of the actual power delivered by the network into an output waveguide to the maximum power which could be delivered into this waveguide by the source itself. However, for the purpose of specifying the noise properties of the amplifier, one could have chosen arbitrarily some other definition of N_{in} because, as we shall see below, these definitions are simply aids in computing from measurements the effective temperature of the noise generated inside the amplifier, and the noise figure obtained with a given choice of definitions is simply a convention for expressing this effective temperature in terms of signal-to-noise ratio.

In this paper, we consider a transmission-type network (a parametric amplifier or a mixer) in which an input signal at frequency ω_1 mixes with a local oscillator at frequency ω_3 to give outputs at both ω_1 and $\omega_2 = \omega_3 - \omega_1$ (see Fig. 1). There are four sources of noise in such a network.

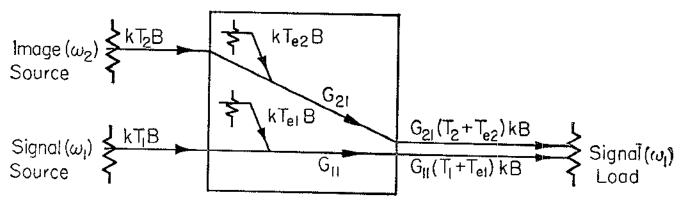

Fig. 1.

1) Noise which flows from the external signal source into the network in the band at frequency ω_1, is amplified, and then flows out at the same frequency;
2) Noise which flows from the external source or image load into the network in the ω_2 band, is frequency converted by mixing with the local oscillator (ω_3), and then flows out in the signal band (ω_1);
3) Noise which is generated inside the amplifier in the frequency band at ω_1 and then flows out at the same frequency;

[5] H. Heffner, "Masers and parametric amplifiers," *Microwave J.*, vol. 2, p. 37; March, 1959.
[6] S. B. Cohn, "The noise figure muddle," *Microwave J.*, vol. 2, pp. 7–11; March, 1959.
[7] "IRE Standards on electron devices—methods of measuring noise," Proc. IRE, vol. 41, pp. 890–893; July, 1953.
[8] "IRE Standards on measuring noise in linear two ports, 1959," Proc. IRE, vol. 48, pp. 60–68; January, 1960.

4) Noise which is generated inside the amplifier in the frequency band at ω_2, is frequency converted, and then flows out in the ω_1 band.

The internally generated noise, 3) and 4), may have its origins in the internal losses of the amplifier, in the local oscillator, in other images of the signal (*e.g.*, $\omega_4 = \omega_3 + \omega_1$), or in other mechanisms internal to the amplifier.[2,3]

If we define G_{11} as the power gain for input and output at frequency ω_1, and G_{21} as the power gain for frequency conversion from an input at ω_2 to an output at ω_1, the total noise power output of the network at frequency ω_1 is

$$N_{\text{out}} = kB(G_{11}T_1 + G_{11}T_{e1} + G_{21}T_{e2} + G_{21}T_2) \quad (3)$$

where

k = Boltzmann's constant,
B = bandwidth of output,
T_1 = noise temperature of external signal source in the ω_1 band,
T_2 = noise temperature of external load or source in the ω_2 band,
T_{e1} and T_{e2} = the effective input noise temperatures of the internal ω_1 and ω_2 circuits, respectively. These are the noise temperatures of resistors which when connected to the appropriate input terminals of the network would result in the same noise output as the noise generated internal to the network.

For convenience, let us define a total effective internal noise temperature by

$$T_e = T_{e1} + \frac{G_{21}}{G_{11}} T_{e2}. \quad (4)$$

This is the noise temperature of a resistor which when connected only to the ω_1 input terminals would result in the same output noise power as the noise generated internal to the network.

A. Single-Sideband Operation (Input at ω_1 Only)

As a first example, let us assume that signal power flows into the network only at ω_1, that only the ω_1 input terminals are connected to a noise source of effective temperature T_1, and that the ω_2 circuit is connected only to load at temperature T_2. Then the output noise at frequency ω_1 is

$$N_{\text{out}} = kB(G_{11}T_1 + G_{21}T_2 + G_{11}T_e). \quad (5)$$

The noise factor is

$$F_{\text{SSB}} = \frac{N_{\text{in}}}{N'_{\text{out}}} \frac{N_{\text{out}}}{N_{\text{in}}}$$

$$= \frac{N_{\text{in}}}{N'_{\text{out}}} \frac{kB(G_{11}T_1 + G_{21}T_2 + G_{11}T_e)}{kBT_1} \quad (6)$$

where T_1 must be taken as $T_0 = 290°\text{K}$. Since

$$\frac{N_{\text{in}}}{N'_{\text{out}}} = \frac{1}{G_{11}},$$

we have

$$F_{\text{SSB}} = 1 + \frac{T_e}{T_0} + \frac{G_{21}}{G_{11}} \frac{T_2}{T_0}. \quad (7)$$

B. Double-Sideband Radiometer Operation (Incoherent Inputs at ω_1 and ω_2)

On the other hand, if a broad-band noise source of temperature T_1 is used to feed equal power in at both ω_1 and ω_2, the noise out is

$$N_{\text{out}} = kB(G_{11}T_1 + G_{21}T_1 + G_{11}T_e), \quad (8)$$

and the noise incident on the amplifier is

$$N_{\text{in}} = 2kT_1B \quad (9)$$

where equal amounts of power flow into the signal and image input terminals. The noise factor is, therefore,

$$F_{\text{DSB}} = \frac{N_{\text{in}}}{N'_{\text{out}}} \frac{kB(G_{11}T_1 + G_{21}T_1 + G_{11}T_e)}{2kBT_1} \quad (10)$$

where T_1 must be taken as $T_1 = T_0 = 290°\text{K}$ in (10). In radiometer operation, the incident signal is equivalent to the thermal noise

$$N_{\text{in}} = 2kBT_s \quad (11)$$

where equal amounts of power flow into the signal and image input terminals. The output signal is then

$$N'_{\text{out}} = kT_sBG_{11} + kT_sBG_{21} \quad (12)$$

so that the noise factor is

$$F_{\text{DSB}} = \frac{2}{G_{11} + G_{21}} \frac{G_{11}T_0 + G_{21}T_0 + G_{11}T_e}{2T_0}$$

$$= 1 + \frac{G_{11}}{G_{11} + G_{21}} \frac{T_e}{T_0}. \quad (13)$$

For the special case where $G_{11} = G_{21}$, we have

$$F_{\text{DSB}} = \tfrac{1}{2}F_{\text{SSB}}, \quad (14)$$

where it has been assumed that for single-sideband operation the image circuit load is at room temperature (*i.e.*, $T_2 = T_0$). These conditions will be satisfied for a quasi-degenerate parametric amplifier at high gain.

For $G_{11} \neq G_{21}$ or for $T_2 \neq T_0$, the radiometer double-sideband noise figure will not be one half the single-sideband value. Even for quasi-degenerate operation (*i.e.*, where ω_1 and ω_2 are almost equal) G_{21} and G_{11} can differ by factors of 10 so that caution should be exercised in using (14).[1]

C. Double-Sideband Coherent Operation (Coherent Inputs at ω_1 and ω_2)

Now suppose that equal amplitude coherent signals flow into the network at frequencies ω_1 and ω_2. Suppose also, that the time origin has been chosen so that the fields at ω_1 and at the pumping frequency are in phase. Then, for a transmission-type parametric amplifier using a nonlinear capacitor, it is shown in Appendix I that, if ϕ is the phase lead of the input ω_2 relative to the input at ω_1, the ratio of S_{out}, the total signal power out at frequency ω_1, to S_{in}, the total available signal power in at frequencies ω_1 and ω_2, is

$$\frac{S_{\text{out}}}{S_{\text{in}}} = \frac{1}{2}\left[G_{11} + G_{21} + 2\sqrt{G_{11}G_{21}}\cos\left(\phi + \frac{\pi}{2}\right)\right]. \quad (15)$$

Since the input and output noise will be the same as in double-sideband radiometer operation, (8), (9), and (15) give the relative input and output signal-to-noise ratios for double-sideband coherent operation of the capacitive parametric amplifier

$$\frac{S_{\text{in}}/N_{\text{in}}}{S_{\text{out}}/N_{\text{out}}} = \frac{G_{11}T_0 + G_{21}T_0 + G_{11}T_e}{\left[G_{11} + G_{21} + 2\sqrt{G_{11}G_{21}}\cos\left(\phi + \frac{\pi}{2}\right)\right]T_0}. \quad (16)$$

For a mixer using a nonlinear resistance, a similar analysis shows that the power gain for coherent double-sideband operation is

$$\frac{S_{\text{out}}}{S_{\text{in}}} = \frac{1}{2}\left[G_{11} + G_{21} + 2\sqrt{G_{11}G_{21}}\cos(\phi + \pi)\right], \quad (17)$$

and

$$\frac{S_{\text{in}}/N_{\text{in}}}{S_{\text{out}}/N_{\text{out}}} = \frac{G_{11}T_0 + G_{21}T_0 + G_{11}T_e}{\left[G_{11} + G_{21} + 2\sqrt{G_{11}G_{21}}\cos(\phi + \pi)\right]T_0}. \quad (18)$$

For the special case of optimum phase (*i.e.*, when the cosine is unity) when also $G_{11} \doteq G_{21}$, we have

$$\frac{S_{\text{in}}/N_{\text{in}}}{S_{\text{out}}/N_{\text{out}}} = \frac{1}{2} + \frac{1}{4}\frac{T_e}{T_0} \quad (19)$$

$$= \frac{1}{4}F_{\text{SSB}} \quad (20)$$

where it has been assumed that $T_2 = T_0$ for operation as a single-sideband amplifier.

From (19), we see that for $T_e < 2T_0$ (a condition which can be achieved in practice) it is possible to obtain a signal-to-noise ratio at the output of the device which is larger than it is at the input. At first thought, this may seem to imply that this device violates the laws of thermodynamics by absorbing thermal noise from the input, but further reflection shows that this is not the case; instead, the device adds noise to the input (because $T_e > 0$) but the amplitude of the noise is not increased proportionally as much as is the amplitude of the signal. This occurs because the coherence of the two input "signals" at ω_1 and ω_2 and the pump at ω_3 allows the signal to be selectively amplified relative to the noise. In order for this improvement in signal-to-noise ratio to occur, it is necessary for the ω_1 and ω_2 input signals to have a certain phase relationship with respect to the pump at ω_3. Since on the average the noise at ω_1 and ω_2 will not have the correct phase relative to the pump (ω_3), the noise will not be amplified as much as is the signal.

III. Measurement of the Noise Factor with a Broad-Band Noise Source

One of the most convenient methods for measuring the noise factor of a network consists of connecting a noise source to the input of the network, and varying the effective temperature of this source so as to double the output noise power of the amplifier.

Let us assume that a matched variable room temperature attenuator is placed in the transmission line between the noise source (with effective temperature βT_0) and the input of the network. Let the ratio of power transmitted to power incident for this attenuator be α.

A. Single-Sideband Noise Factor Measurement

Assume that the input is filtered so that one need consider only power which flows into the network from the noise source in a frequency band of width B at ω_1 and that the ω_2 circuit is connected only to a load at temperature T_2. With the noise source off (*i.e.*, with $\alpha \ll 1$), the input noise power will be

$$(N_{\text{in}})_1 = kT_0B. \quad (21)$$

The resulting output at ω_1 will be

$$(N_{\text{out}})_1 = G_{11}kT_0B + G_{21}kT_2B + G_{11}kT_eB. \quad (22)$$

When the noise source (of temperature βT_0) is turned on, and the attenuator is set at a value α which gives double the noise output, the noise input power is given by

$$(N_{\text{in}})_2 = \alpha k(\beta T_0)B + (1-\alpha)kT_0B. \quad (23)$$

The resulting output power at ω_1 will be

$$(N_{\text{out}})_2 = G_{11}[1 + \alpha(\beta - 1)]kT_0B + G_{21}kT_2B + G_{11}kT_eB \quad (24)$$

$$= 2(N_{\text{out}})_1. \quad (25)$$

Solving (24), (25), and (22) for T_e, we obtain

$$T_e = [\alpha(\beta - 1) - 1]T_0 - \frac{G_{21}}{G_{11}}T_2. \quad (26)$$

If we consider the image load to be a part of the network, then the total effective noise temperature (*i.e.*, the temperature of noise generated both internally and in the image load) will be

$$(T_e)_{\text{total}} = T_e + \frac{G_{21}}{G_{11}} T_2 \tag{27}$$

or using (26),

$$(T_e)_{\text{total}} = [\alpha(\beta - 1) - 1]T_0. \tag{28}$$

From (7), the single-sideband noise factor will therefore be

$$F_{\text{SSB}} = \alpha(\beta - 1). \tag{29}$$

B. Radiometer Double-Sideband Noise Factor Measurement

If the input is not filtered, and power flows into the network in a band of width B at frequency ω_1, and in a band of width B at frequency ω_2, the total input noise power when the noise source is off will be

$$(N_{\text{in}})_1 = 2kT_0B. \tag{30}$$

The resulting output at ω_1 will be

$$(N_{\text{out}})_1 = G_{11}kT_0B + G_{21}kT_0B + G_{11}kT_eB. \tag{31}$$

When the noise source (of temperature βT_0) is turned on and the attenuator is set at a new value α' to double the output noise power, the input noise is

$$(N_{\text{in}})_2 = 2\alpha' k(\beta T_0)B + 2(1 - \alpha')kT_0B, \tag{32}$$

where half the power flows in at each frequency (ω_1 and ω_2), and the resulting output power at ω_1 is

$$(N_{\text{out}})_2 = G_{11}[\alpha'(\beta - 1) + 1]kT_0B + G_{21}[\alpha'(\beta - 1) + 1]kT_0B + G_{11}kT_eB \tag{33}$$

$$= 2(N_{\text{out}})_1. \tag{34}$$

Solving (31), (33), and (34) for T_e, we obtain

$$T_e = \frac{G_{11} + G_{21}}{G_{11}} [\alpha'(\beta - 1) - 1]T_0. \tag{35}$$

From (13), the radiometer double-sideband noise factor will therefore be

$$F'_{\text{DSB}} = \alpha'(\beta - 1). \tag{36}$$

Comparing (36) with (29), we see how confusion can arise if the experimenter does not take care to note whether or not power flows into the network at both the ω_1 and ω_2 bands.

The single-sideband noise factor can be computed from the wide-band noise source measurements by substituting (35) in (7), and noting that $T_2 = T_0$. Doing this, one obtains

$$F_{\text{SSB}} = \left(1 + \frac{G_{21}}{G_{11}}\right) \alpha'(\beta - 1). \tag{37}$$

For the special case where $G_{21} = G_{11}$ (for example, in a quasi-degenerate parametric amplifier at high gain), the measured radiometer double-sideband noise factor is one half the single-sideband value.

IV. Discussion

From Sections II and III, one sees that the principal difference between single-sideband and double-sideband operation lies in whether the image circuit external load is considered as part of the network or as part of the source. For single-sideband operation, this load is included in the amplifier and contributes a term to the total effective noise temperature of the device. For double-sideband operation, this load is a part of the source, and the device cannot be penalized for noise generated by it.

The difference between radiometer and coherent double-sideband operation depends upon whether or not the inputs in the two sidebands are coherent. If they are coherent, then by properly choosing their phases relative to the pump, it is possible to amplify selectively these coherent inputs and increase the output signal-to-noise ratio relative to the input signal-to-noise ratio. It should be noted that the optimum choice of relative phase depends upon whether a nonlinear capacitance, a nonlinear inductance, or a nonlinear resistance is used as the time varying element in the mixing network.

One further possible point of confusion remains with respect to the single-sideband mixer or parametric amplifier: if the signal field is fed only into the network at frequency ω_1 but both the ω_1 and ω_2 input terminals are connected to the antenna, then the total effective noise temperature of the device $(T_e)_{\text{total}}$ [see (27)] will depend upon T_2, the temperature of the input circuit at ω_2. This means that when used with a low temperature source the noise figure of a single-sideband mixer circuit will be lower than the value computed from the measurements of Section III.

From the section on measurements, one sees that the confusion as to what parameter is actually being measured is greatly reduced if it is realized that the noise tube technique is actually a means for determining the total effective internal noise temperature T_e, and, that from this quantity, the noise figure can be computed for each specific type of operation.

Appendix I

Derivation of Gain Expression for Coherent Double-Sideband Operation

Using the definitions as indicated in Fig. 2, one can derive the expression for the midband gain of an inverting transmission-type parametric amplifier for coherent double-sideband operation as follows.[2]

Assume a current generator I_1 is connected to the ω_1 input of the amplifier and that a current generator I_2 is connected to the ω_2 input (See Fig. 2). Let V_{11} be the voltage at frequency ω_1, developed as a result of I_1, and let V_{21} be the voltage at frequency ω_1 developed as a result of I_2.

Letting $\alpha = g/g_1$ in Heffner,[3] we have

$$V_{11} = \frac{I_1}{g_1(1 - \alpha)}.$$

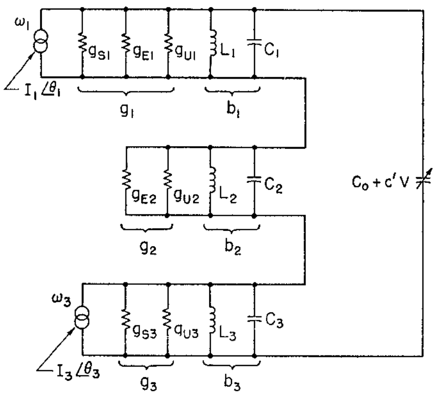

Fig. 2.

From the treatment of Heffner,[3] one can show that

$$V_{21} = -\frac{j\omega_1 c' V_3}{g_1 g_2(1-\alpha)} I_2^*.$$

The total signal power delivered to g_{E1}, the external load in the ω_1 circuit, will be

$$S_0 = (V_{11} + V_{21})(V_{11}^* + V_{21}^*)G_{E1}.$$

Assuming $I_2 = Ie^{j(\omega_2 t+\phi)}$, and $I_1 = Ie^{j\omega_1 t}$, and assuming the phase of the pump to be zero, this can be rewritten as

$$S_0 = G_{11}S_1 + G_{21}S_2 + \frac{2g_{E1}\omega_1 c' V_3 I^2}{g_1^2 g_2(1-\alpha)^2}\cos\left(\phi + \frac{\pi}{2}\right),$$

where

$$S_1 = \frac{|I_1|^2}{4g_{S1}} = \text{signal power incident at frequency } \omega_1$$

$$S_2 = \frac{|I_2|^2}{4g_{S2}} = \text{signal power incident at frequency } \omega_2$$

$$G_{11} = \frac{4g_{E1}g_{S1}}{g_1^2(1-\alpha)^2}$$

$$G_{21} = \frac{\omega_1}{\omega_2}\,\frac{4g_{S2}g_{E1}}{g_1 g_2}\,\frac{\alpha}{(1-\alpha)^2}.$$

Since for small signals

$$\alpha \doteq \frac{\omega_1\omega_2(c')^2 V_3^2}{g_1 g_2},$$

we obtain

$$S_0 = G_{11}S_1 + G_{21}S_2 + 2\sqrt{G_{11}S_1G_{21}S_2}\cos\left(\phi + \frac{\pi}{2}\right).$$

The incident signal power is given by

$$S_i = S_1 + S_2.$$

But, for equal input couplings $S_1 = S_2$, so that the gain for coherent inputs at ω_1 and ω_2 with output at ω_1 is

$$G_{\text{DSB}} = \frac{S_0}{S_i} = \frac{1}{2}\left[G_{11} + G_{21} + 2\sqrt{G_{11}G_{21}}\cos\left(\phi + \frac{\pi}{2}\right)\right].$$

A similar analysis could be carried out for a nonlinear resistance mixer; it would then be found that the factor $\cos(\phi + \pi/2)$ in the preceding should be replaced by the factor $\cos(\phi + \pi)$.

Duplexing Systems at Microwave Frequencies*

A. F. HARVEY†

Summary—**The paper reviews the various methods of duplexing at microwave frequencies. General principles, including the use of passive and solid-state devices, are first discussed. The characteristics of gaseous-discharge duplexing tubes of both self- and externally-excited types are examined and data for typical examples given. The various arrangements of discharge tube duplexers and methods of measuring their performance are described. The survey concludes with a bibliography.**

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List of Principal Symbols

B_0 = applied magnetic flux density, weber/m^2.
c = speed of light in vacuo $= (\epsilon_0\mu_0)^{-1/2} = 3\times10^8$ m/sec.
D_a = ambipolar diffusion coefficient, m^2/sec.
e = charge on electron $= 1.60207\times10^{-19}$ coulomb.
h_a = attachment probability.
m = mass of electron $= 9.1085\times10^{-31}$ kg.
N = density of electrons, m^{-3}.
N_L = Loschmidt's constant $= 2.687\times10^{25}$ m^{-3} atm^{-1}.